Narrowband Filters on Astronomical Telescopes

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Contents

This document contains the rational for the use of narrowband interference filters on astronomical telescopes. It shall enable the user to choose the optimal filter for his telescope.

1 Optical Interference Filters

Optical interference filters consist of optical glass on which thin transparent layers of materials, with different refractive indices are deposited. The partial reflection at the layer boundaries lead to interference of the refracted and incident light. The dielectric layer structure is designed in such a way that the filter obtains the desired transmission or reflection behavior.

2 Interference Filters on the Telescopes (in the Beam)

2.1 Angle Dependence

The interference effect depends on the angle of incidence of the light path. If the filter is tilted in the parallel beam path, or if the light occurs at an angle of incidence other than 0° (vertical), a change of the filter characteristic occurs. The filter bandpass undergoes a blue shift, i.e., it is shifted to shorter wavelengths. This can be approximated by the following formula:

λ_θ	Wavelength under angle of incidence ϕ
λ_0	Wavelength at angle of incidence 0°
θ	Angle of incidence
n_0	refractive index of the incident medium (air)
n_{eff}	effective refractive index of the filter

$$\lambda_\phi = \lambda_0 \sqrt{1 - \left(\frac{n_0 \ sin\theta}{n_{eff}}\right)^2} \tag{1}$$

The blue shift is directly proportional to the wavelength as well as to the angle of incidence. Therefore, it is stronger at longer wavelengths and larger angles of incidence.

2.2 Beam and Obstruction

On an astronomical telescope, the incident light passes through the interference filter at a variety of angles. The smaller the eyepiece ratio (focal length divided by eyepiece), the larger are the differences of the angles of incidence of the edge and central rays of the light beam.

$$f/\# = \frac{f}{D} \tag{2}$$

Figure 1 on the following page shows the different angles of incidence of the central and edge beams.

- θ_{min}: Angle of incidence of the central rays (smallest angle of incidence at the filter).
- θ_{max}: Angle of incidence of the edge rays (largest angle of incidence at the filter).

The effective bandpass $T(\lambda)$ of an interference filter at the real telescope is thus created by superimposing of the angle-dependent single bandpasses $T(\lambda, \theta)$ at all different angles of incidence.
Mathematically, the angle-dependent transmission function, weighted with the circumference of the circle $U(\theta)$, is integrated from the angle of incidence of the central beam θ_{min} to the angle of incidence of the edge beam θ_{max}.

$$T_{eff}(\lambda) = \int_{\theta_{min}}^{\theta_{max}} T(\lambda, \theta) U(\theta) \, d\theta \tag{3}$$

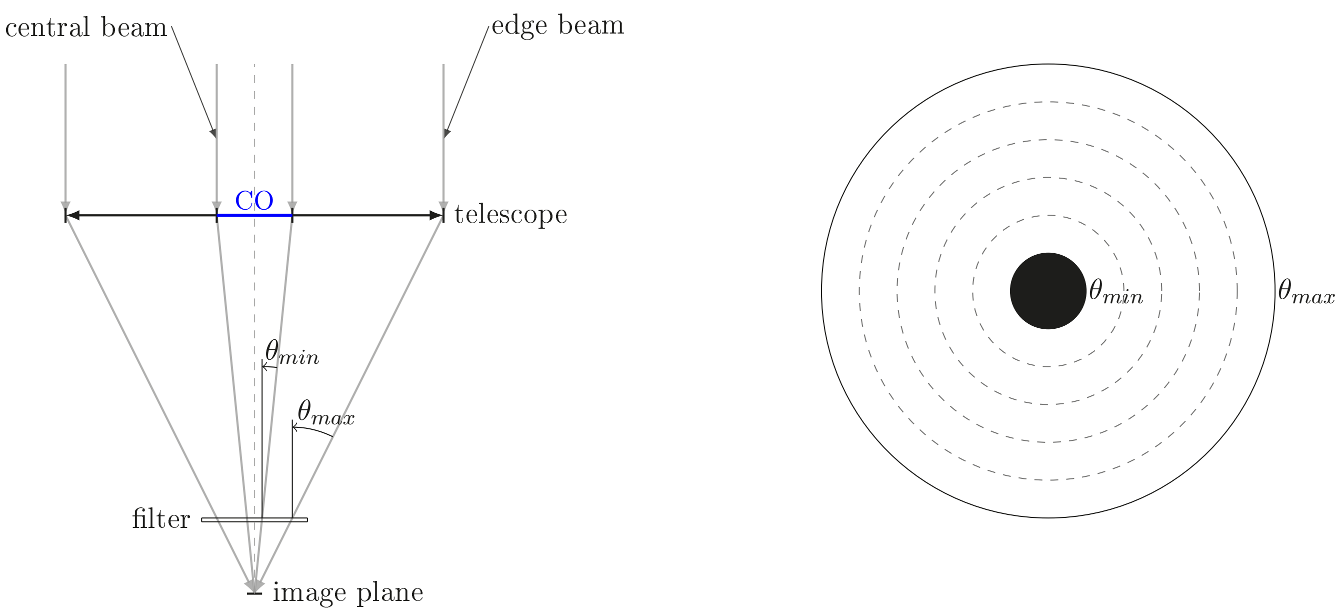
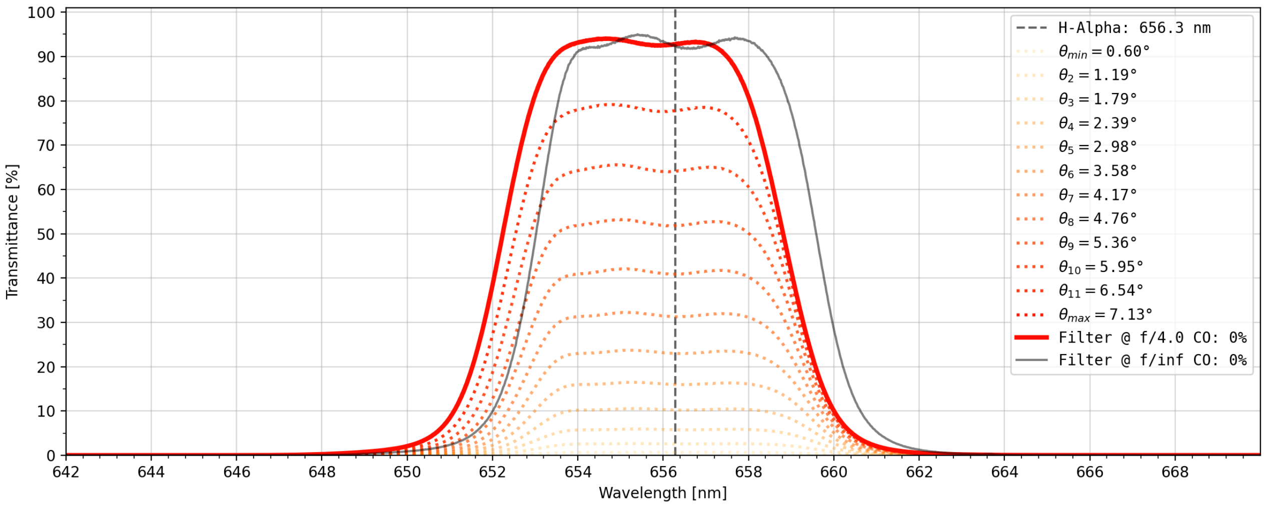


Figure 1: left: Telescope light beam with central obstruction (CO). Right: Top view of telescope aperture with partial rings (θ_{min} to θ_{max}).

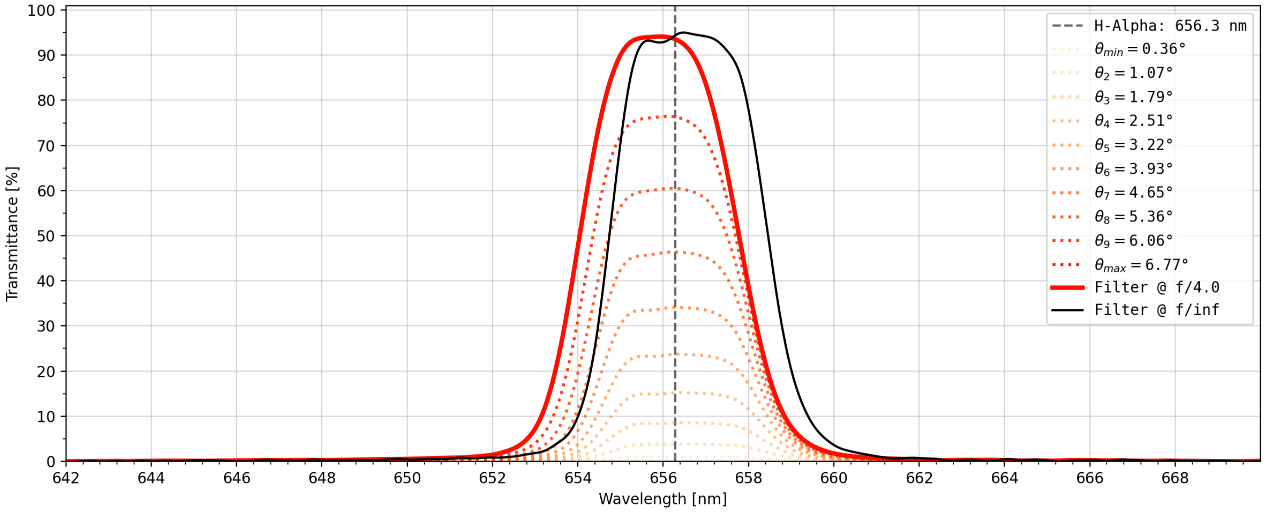
Figure 2 on the next page shows two H-alpha filters with different bandpass widths at an aperture ratio of $f/4$. The telescope aperture is divided into 12 partial rings of equal width and the respective individual bandpass of each partial ring is calculated using the average angle of incidence. The effective bandpass of the entire aperture is generated by adding the individual bandpasses. The figure shows in dotted lines the contributions of the individual bandpasses $T(\theta)$ (inside to outside) to the effective bandpass T_{eff}. As the area of the partial rings grows quadratically with increasing radius, the outer partial rings are weighted accordingly to their larger area, They therefore have a stronger influence on the total effective bandpass than the inner. smaller partial rings.

2.3 Large Aperture Ratios ($> f/4$)

The angular dependence of the interference filter has virtually no significance for telescopes with a large focal ratio ($> f/4$), given that the angular difference and thus the blue shift are relatively small. Figure 2a and 2b show H-alpha filters with a half-width of 6.5 nm and 3.5 nm, respectively, at $f/4$. The blue shift is so small that the filter still has almost maximum transmission at $H\alpha = 656.3\,nm$.



(a) Bandpass of an H-alpha 6.5nm Narrowband filter in the parallel beam path (f/inf) as well as at a focal ratio of $f/4$. The blue shift has no influence on the maximum transmission at $H\alpha$.



(b) Bandpass of an H-alpha 3.5nm Ultra-Narrowband filter in the parallel beam path (f/inf) and at a focal ratio of $f/4$. The blue shift has no influence on the maximum transmission at $H\alpha$.

Figure 2: H-Alpha 6.5nm Narrowband and 3.5nm Ultra-Narrowband (centred on the $H\alpha$ emission line) filters in the f/4 beam path. The dotted graphs show the cumulative contribution of the single bandpasses of 12 subrings of the telescope aperture with the respective mean angles of incidence θ of the partial ring. As the angle θ increases, both the absolute blue shift as well as the area of the partial ring increases and thus its contribution to the effective total bandpass.

2.4 Small Aperture Ratios ($< f/4$)

For telescopes with a small focal ratio, however, the blue shift is very relevant, because here the angular differences are larger. This leads to a larger blue shift ($\sim \theta_{max}$), as well as to a stronger enlargement of the bandpass ($\sim \Delta\theta = \theta_{max} - \theta_{min}$). Figures 3 to 4b on the next page, illustrate that the maximum transmission of a narrowband filter centred on the target wavelength would be significantly reduced by the blue shift without suitable compensation (cf. section 3) would be significantly reduced.

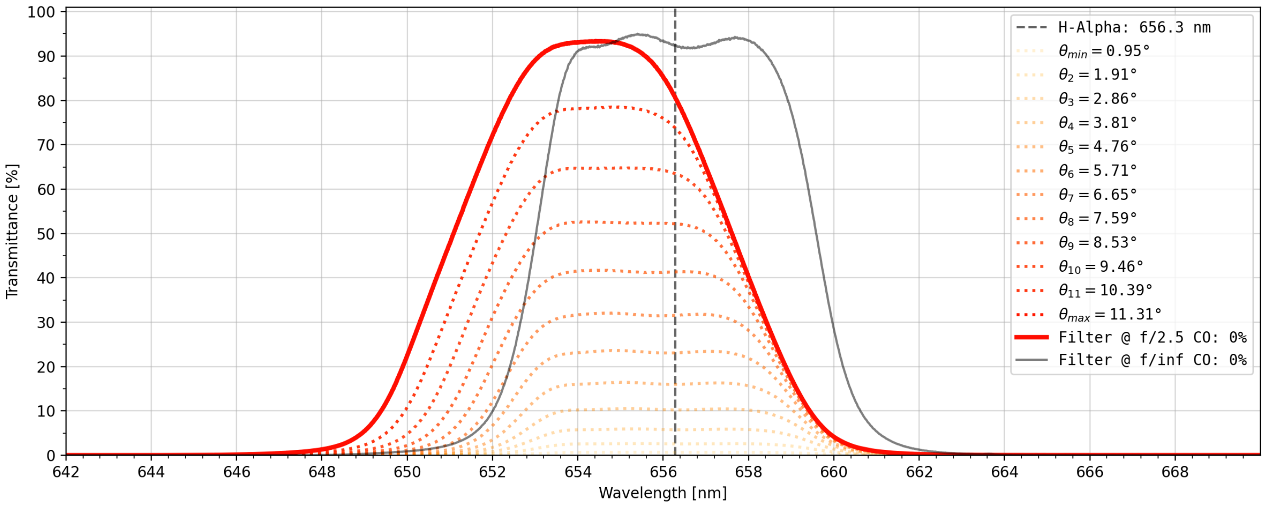
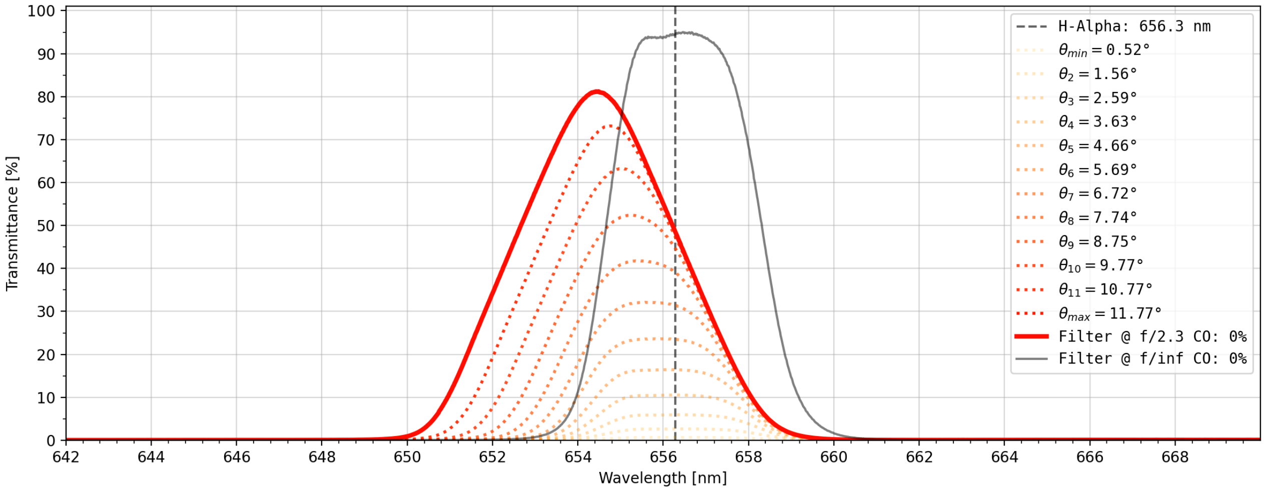
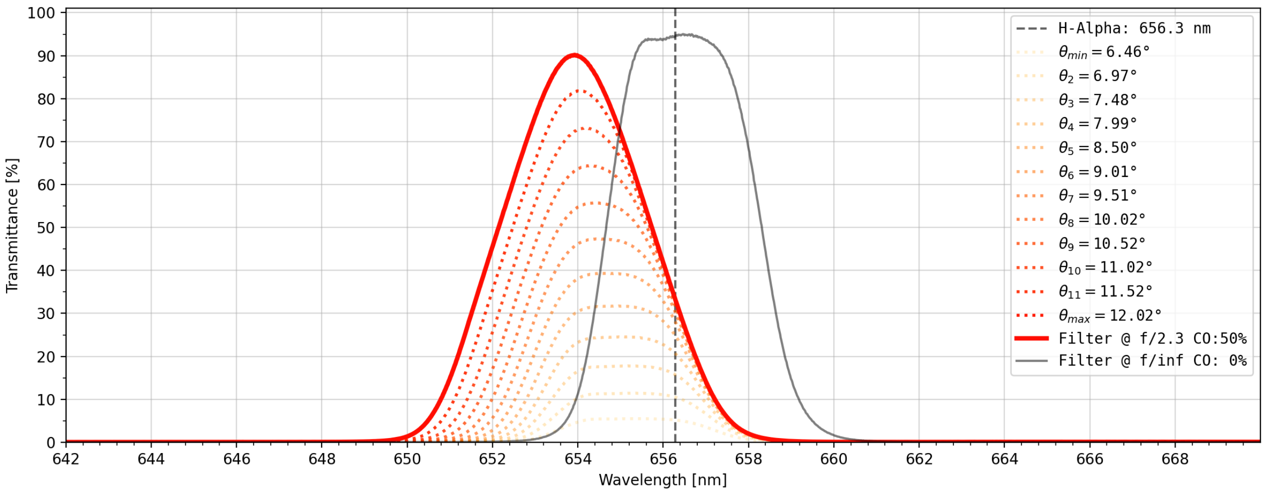


Figure 3: Effective bandpass of a H-Alpha 6.5nm Narrowband filter centred on the $H\alpha$ emission line in the parallel beam path (f/inf) and at an aperture ratio of $f/2.5$. The dotted graphs show the cumulative contribution of the single bandpasses of 12 sub-rings of the telescope aperture with the respective angles of incidence θ. The larger the partial ring, the larger the blue shift and its contribution to the effective bandpass. Due to the blue shift, the maximum transmission at $H\alpha$ would only be approx. 80%.



(a) Effective bandpass of an H-Alpha 3.5nm Ultra-Narrowband filter centred on $H\alpha$ in the parallel beam path (f/inf) as well as at an aperture ratio of $f/2.3$. Due to the blue shift, the maximum transmission at $H\alpha$ would only be approx. 50%.



(b) Effective bandpass of an $H\alpha$ centred H-Alpha 3.5nm Ultra-Narrowband filter in the parallel beam path (f/inf) as well as at a focal ratio of $f/2.3$ with a central obstruction of 50% (of the diameter). The central obstruction leads to a stronger blue shift with a narrower half-value width. Due to the blue shift, the maximum transmission at $H\alpha$ would only be approx. 30%.

Figure 4: H-Alpha 3.5nm Ultra-Narrowband Filter filter (centred on the $H\alpha$ emission line) in the $f/2.3$ beam path. The dotted graphs show the cumulative contribution of the single bandpasses of 12 subrings of the telescope aperture with the respective angles of incidence θ. The larger the partial ring, the larger the blue shift and its contribution to the effective bandpass. A narrowband filter centred on the $H\alpha$ emission line, with a small aperture ratio and, if necessary, central obstruction would have a lower transmission due to the blue shift.

3 Pre-shifted Filter to Compensate the Blueshift

Telescopes with a small focal ratio require filters that are factory-fitted with a redshift to compensate the blue shift (so-called p*re-shift*) Baader-Planetarium GmbH offers for *Narrowband* filters, two further categories of so-called high-speed filters with pre-shift for different aperture ratios:

6.5nm Narrowband filters are offered in two categories:

Filter name	Optimal aperture ratio
Narrowband	$> f/3.4$
Highspeed f/2	$\leq f/3.4$

Table 1: 6.5nm **Narrowband** categories by aperture ratio

3.5nm/4nm Ultra-Narrowband filters are offered in three categories:

Filter name	Optimal aperture ratio
Ultra-Narrowband	$> f/3.4$
Ultra-Highspeed f/3	$f/2.3 - f/3.4$
Ultra-Highspeed f/2	$< f/2.3$

Table 2: 3.5nm bzw. 4nm **Ultra-Narrowband** categories by aperture ratio

Figures 5 to 10 show the effective bandpass of H-alpha, SII and OIII *Narrowband* filters (Figs. 5 to 7) with 6.5 nm half-width and *Ultra-Narrowband* (Figs. 8 to 10) filters with 3.5 nm or 4 nm half-width at the appropriate focal ratios.

3.1 H-Alpha 6.5nm Narrowband

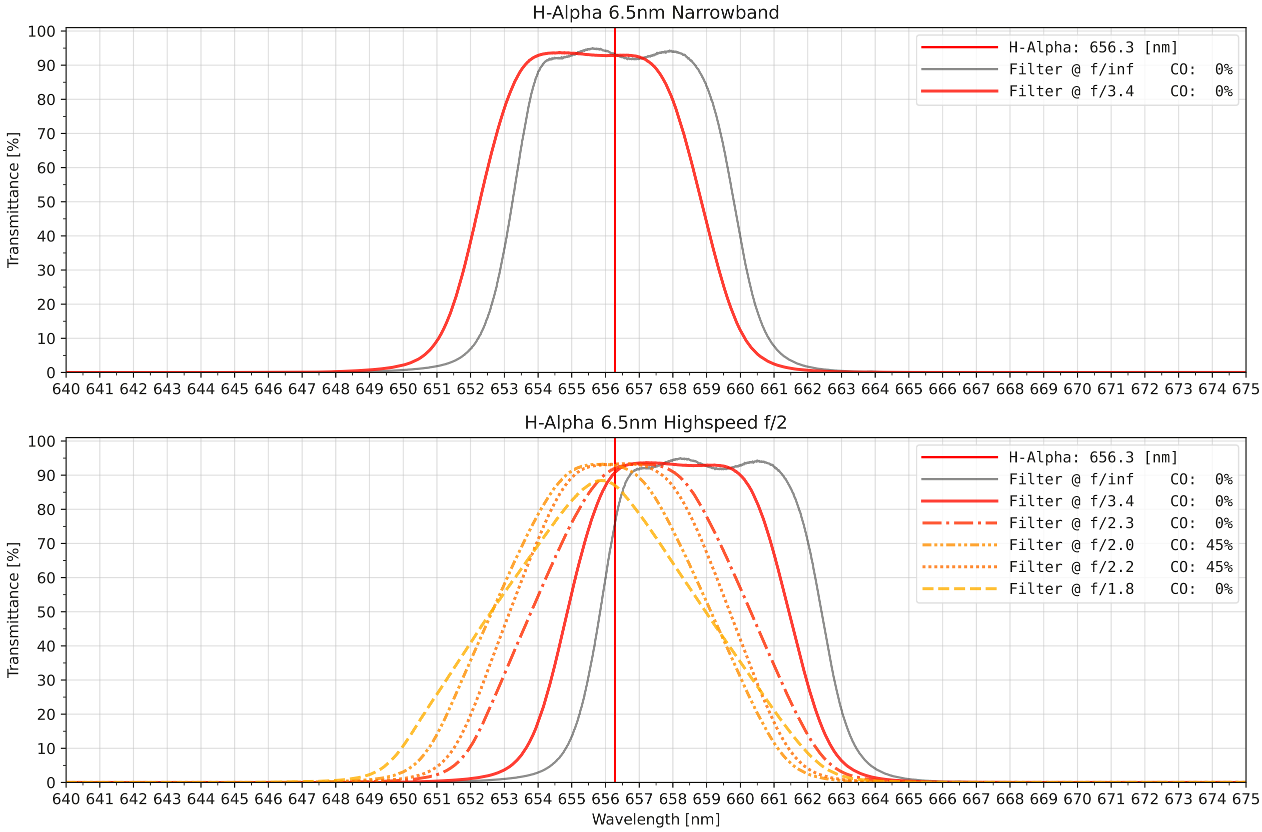


Figure 5: Effective bandpass of the **H-Alpha 6.5nm Narrowband** (top) and **H-Alpha 6.5nm Highspeed f/2** (bottom) filters at different focal ratios and central obstruction **CO** (**C**entral **O**bstruction) as a percentage of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

3.2 SII 6.5nm Narrowband

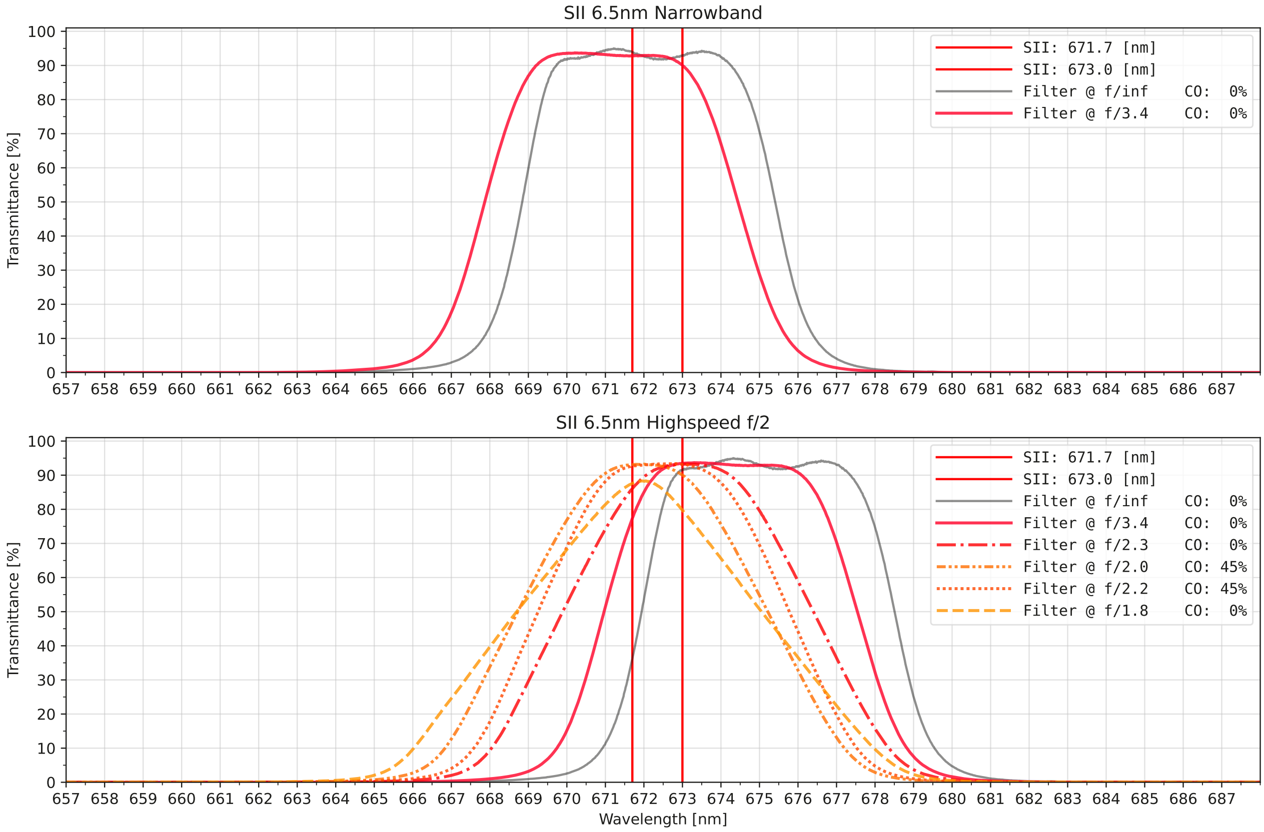


Figure 6: Effective bandpass of the **SII 6.5nm Narrowband** (top) and **SII 6.5nm Highspeed f/2** (bottom) filters at different focal ratios and central obstruction **CO** = (**C**entral **O**bstruction) as a percentage of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

3.3 OIII 6.5nm Narrowband

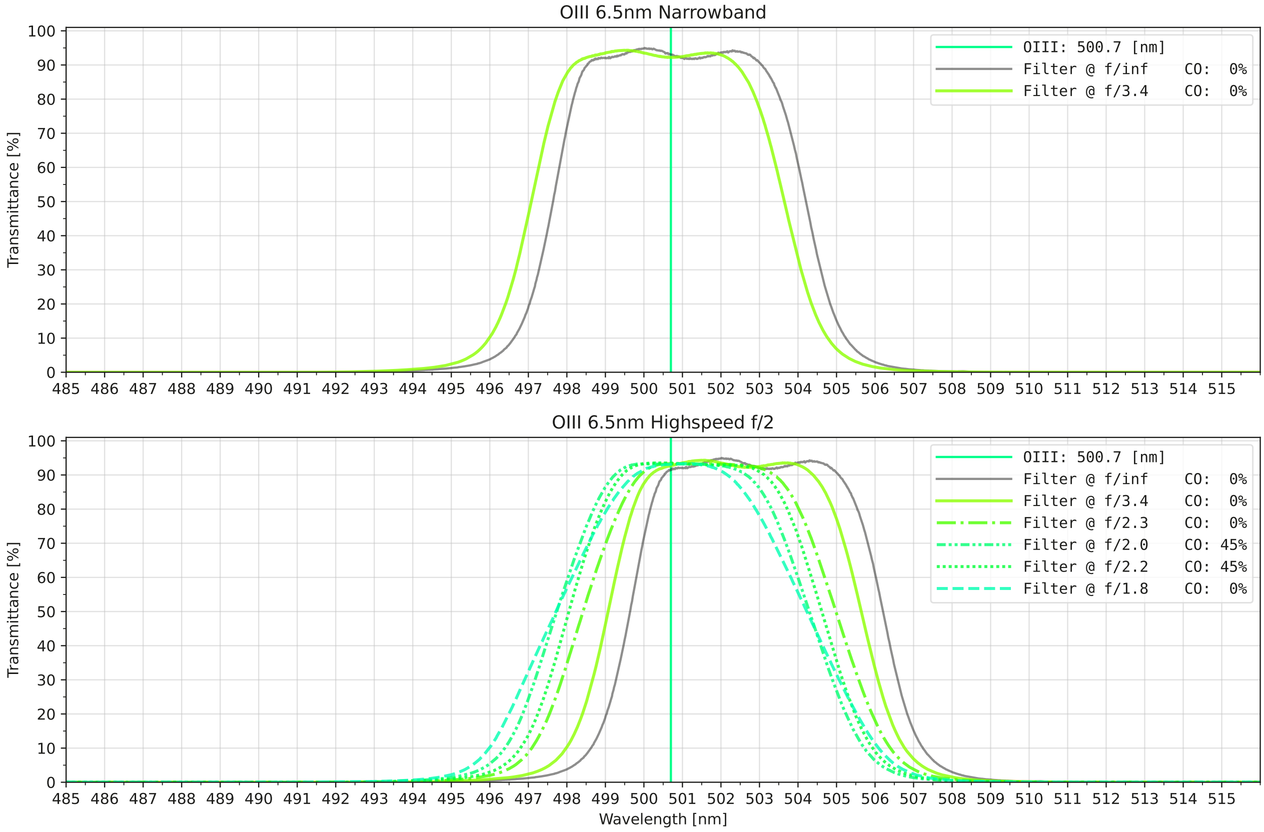


Figure 7: Effective bandpass of the **OIII 6.5nm Narrowband** (top) and **OIII 6.5nm Highspeed f/2** (bottom) filters at different focal ratios and central obstruction **CO** = (**C**entral **O**bstruction) as a percentage of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

3.4 H-Alpha 3.5nm Ultra-Narrowband

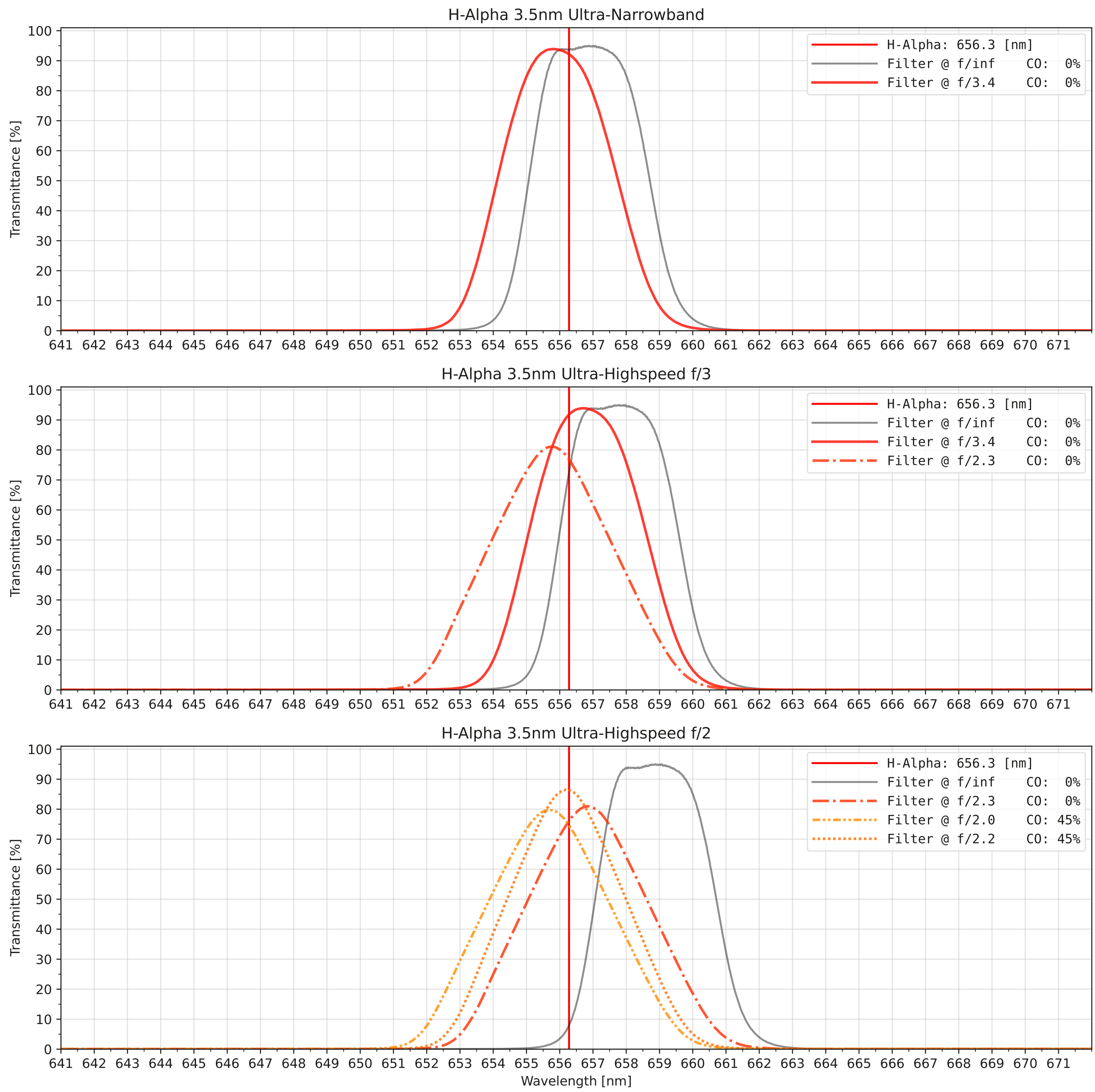


Figure 8: Effective bandpass of an **H-Alpha 3.5nm Ultra-Narrowband**, **H-Alpha 3.5nm Ultra-Highspeed f/3** as well as **H-Alpha 3.5nm Ultra-Highspeed f/2** filter at different focal ratios and central obstruction (**CO**) in percent of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

3.5 SII 4nm Ultra-Narrowband

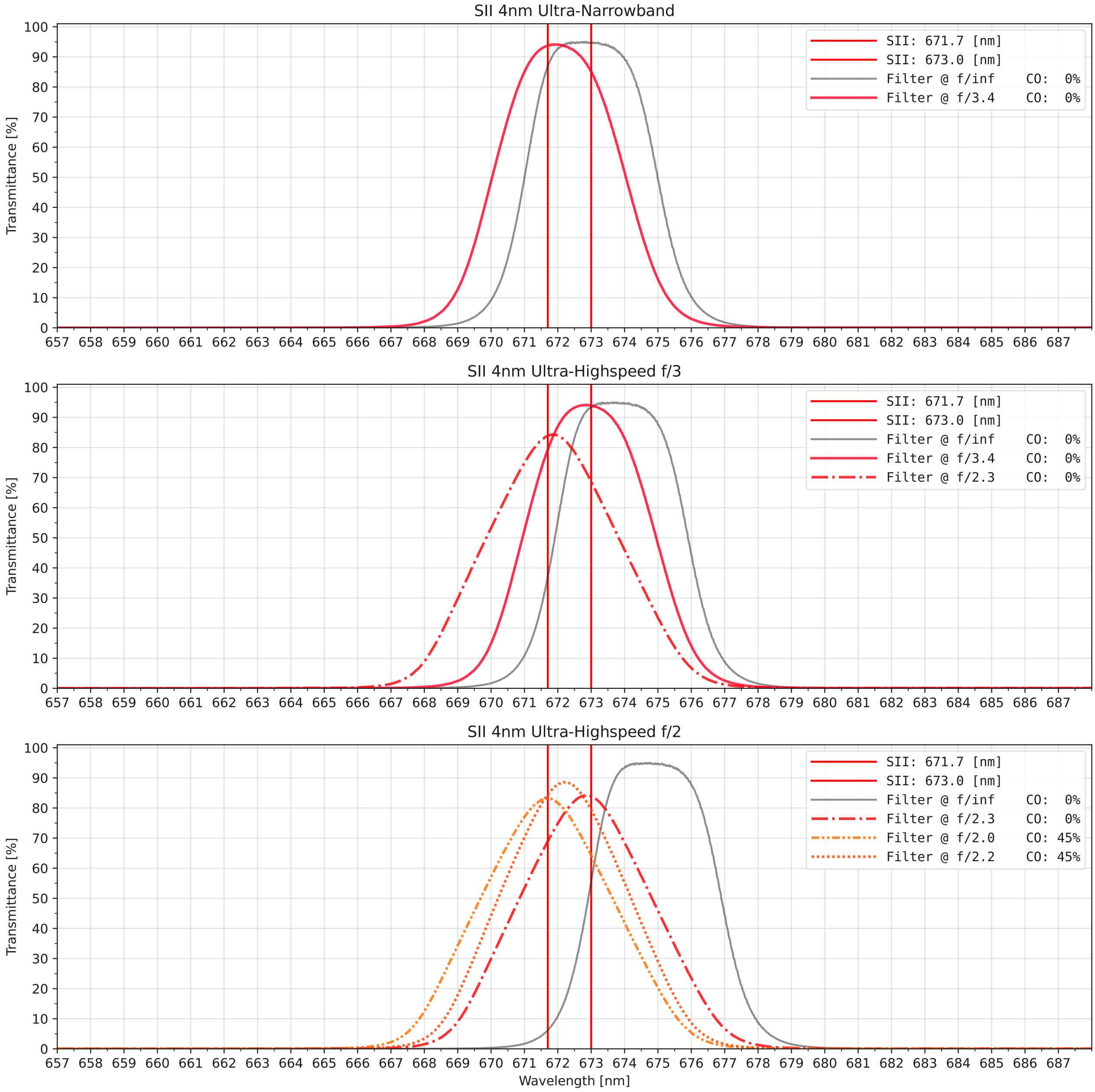


Figure 9: Effective bandpass of an **SII 4nm Ultra-Narrowband**, **SII 4nm Ultra-Highspeed f/3** as well as **SII 4nm Ultra-Highspeed f/2** filter at different focal ratios and central obstruction (**CO**) in percent of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

3.6 OIII 4nm Ultra-Narrowband

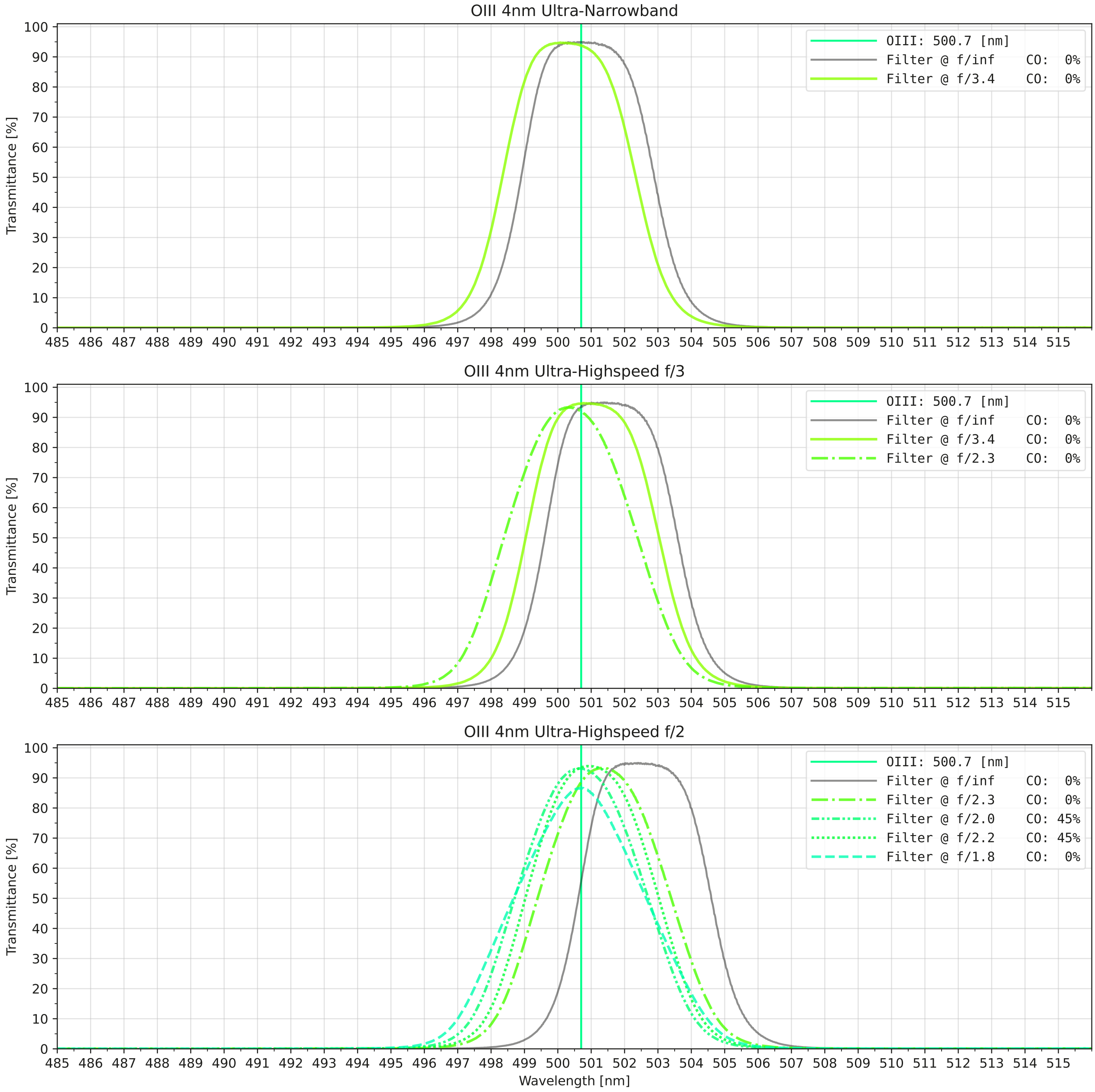


Figure 10: Effective bandpass of an **OIII 4nm Ultra-Narrowband**, **OIII 4nm Ultra-Highspeed f/3** as well as **OIII Ultra-Highspeed f/2** filter at different focal ratios and central obstruction (**CO**) in percent of the aperture diameter (not the area!). (**f/inf** = parallel beam path)

4 Narrowband or Ultra-Narrowband?

In general, the use of narrowband filters serves the purpose of improving the signal-to-noise ratio (SNR). Several factors contribute to the signal-to-noise ratio:

signal	S
light-pollution / skyfog	LS
dark-current	DC
read-noise	RN

Table 3: SNR Beiträge

The signal-to-noise ratio is formed by the quotient of the signal (of the object of interest) by the square root of the noise components:

$$SNR = \frac{S}{\sqrt{S + LS + DC + RN^2}} \tag{4}$$

The readout noise is squared, as it is already given by the manufacturer as an RMS value (*root-mean-square*), thus as the square root of the sums of the individual factors contributing to the readout noise.

To improve the signal-to-noise ratio, one has a few options:

Aim	Action
Direct improve of SNR	· stacking of single exposures (*stacking*).
Reduce dark current	· cool camera
	· Shorten exposure time (via smaller focal ratio)
Reduce read-out noise	· Cool camera
	· Camera settings (gain, ISO etc.)
	· stacking fewer single exposures
Reducing light pollution	· Improve observation conditions (location, weather, moon phase)
	· Use optical filters

Table 4: SNR verbessern

- Dark current and readout noise are characteristics of the camera that can only be influenced to a limited extent (cooling, exposure times, GAIN settings, etc.).
- A better observation site is of course always the first choice, but often difficult to achieve.
- Adding single exposures (so-called stacking) is an effective way to improve the signal-to-noise ratio. However, the ratio of the SNR improvement to the number of single images N is quadratic ($SNR \sim \sqrt{N}$). This means that for an improvement in SNR by a factor of 2, 4 single exposures are necessary, for an improvement by a factor of 4, 16 single exposures are necessary, etc. Furthermore, the readout noise squares for each frame into the SNR calculation. This means that one should first try to expose as long as possible before extending the observation time further by adding frames.

- This is where the narrowband filters in particular come into play. They improve the SNR directly by filtering out the uninteresting spectral components. Irrespective of light pollution, their use also makes significantly longer exposure times possible, since the dynamic range of the camera can be fully exploited for the actual signal. This in turn reduces the amount of readout noise when adding individual images, as you can work with fewer individual exposures and longer exposure times.

The narrower the bandpass of the filter, the better the signal-to-noise ratio. However, the relationship between filter half-width and SNR is not linear. The ratio of the SNR of two filters with different half-widths can be estimated as follows for the spectral range of the bandpass. (For derivation and assumptions, see Appendix A on page 23)

$$\frac{SNR_{FilterA}}{SNR_{FilterB}} = \sqrt{\frac{FWHM_{FilterB}}{FWHM_{FilterA}}} \tag{5}$$

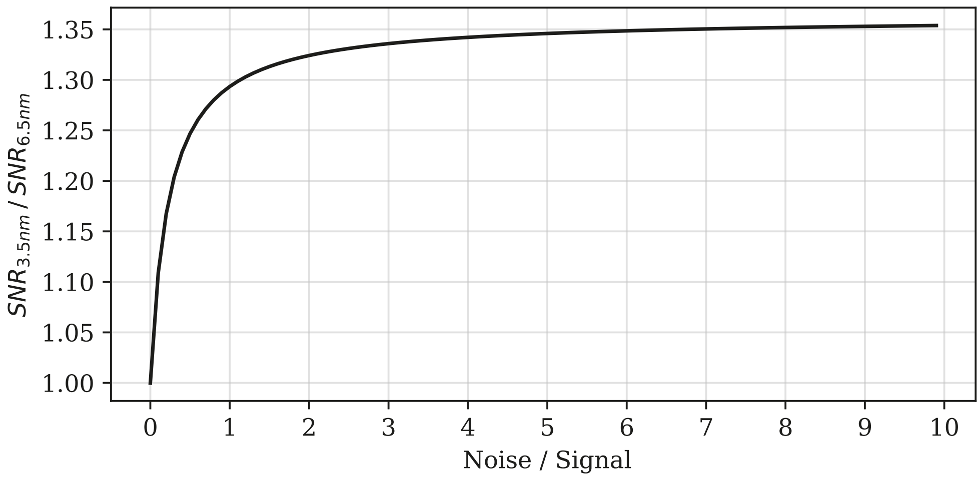


Figure 11: SNR comparison of a **3.5nm Ultra-Narrowband** (filter A) to a **6.5nm Narrowband** filter (filter B): $\frac{SNR_{3.5nm}}{SNR_{6.5nm}}$. The ratio converges towards ≈ 1.4 with increasing light pollution.

Narrowband If you plan to use the filter on several telescopes with different focal ratios, the wider *6.5nm Narrowband* filter is useful, as it will usually work on all your telescopes. In section 5 you can graphically determine the optimal filter for different telescopes.

Ultra-Narrowband Especially in the case of strong light pollution, or for the resolution of the finest details the narrower *3.5nm Ultra-Narrowband* filter is the better choice.

5 Filter Selection According to Aperture Ratio and Central Obstruction

As shown, the central obstruction, in addition to the aperture ratio, has a large influence on the blue shift of a narrowband filter. Selecting the filter only according to the aperture ratio would be a simplification. The following figures 15 to 17 show the suitability of the different filter categories for all relevant combinations of aperture ratio and obstruction. Although temperature plays a subordinate role and is therefore not discussed in detail here, the graphs allow the selection of the filter with further consideration of the expected average temperature of the observation site.

To select the optimal filter for your telescope, proceed as follows:

- Search for the appropriate image depending on the desired filter wavelength:

H-Alpha	6.5 nm	Narrowband	fig. 12 on the next page
SII	6.5 nm	Narrowband	fig. 13 on page 18
OIII	6.5 nm	Narrowband	fig. 14 on page 19
H-Alpha	3.5 nm	Ultra-Narrowband	fig. 15 on page 20
SII	4 nm	Ultra-Narrowband	fig. 16 on page 21
OIII	4 nm	Ultra-Narrowband	fig. 17 on page 22

- Select the graph for the expected ambient temperature at the observation site.
- Select their aperture ratio on the horizontal X-axis.
- Select their relative obstruction on the vertical Y-axis (as a percentage of the diameter, not the area: *diameter of the central obstruction / diameter of the telescope aperture* x *100%*. All refractors are on the X-axis as they have no central obstruction).
- The colour of the area in which your telescope belongs, leads to the optimal filter category for your application.

5.1 H-Alpha 6.5nm Narrowband Filter Selection

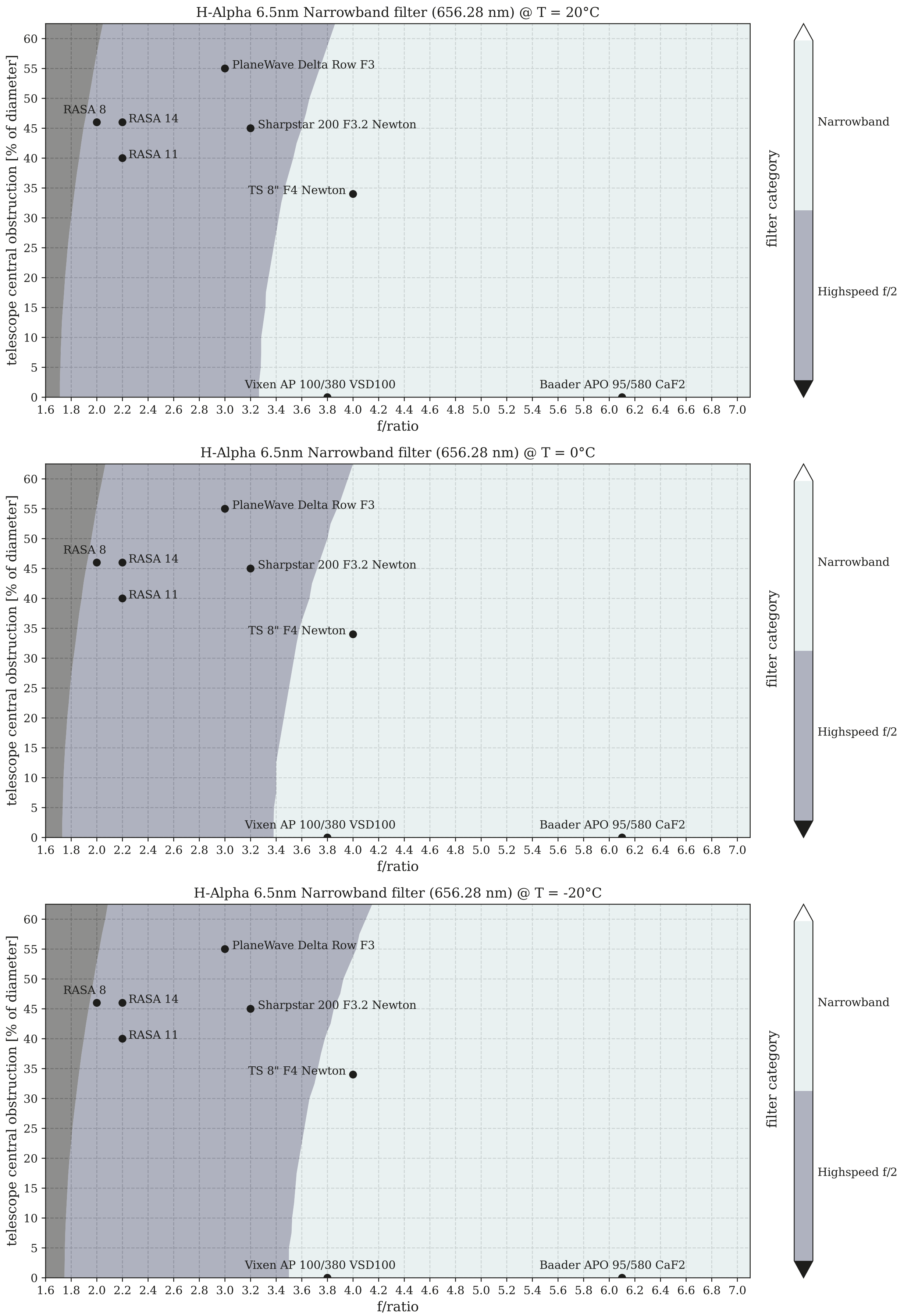


Figure 12: Filter selection guide for **H-Alpha 6.5nm Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

5.2 SII 6.5nm Narrowband Filter Selection

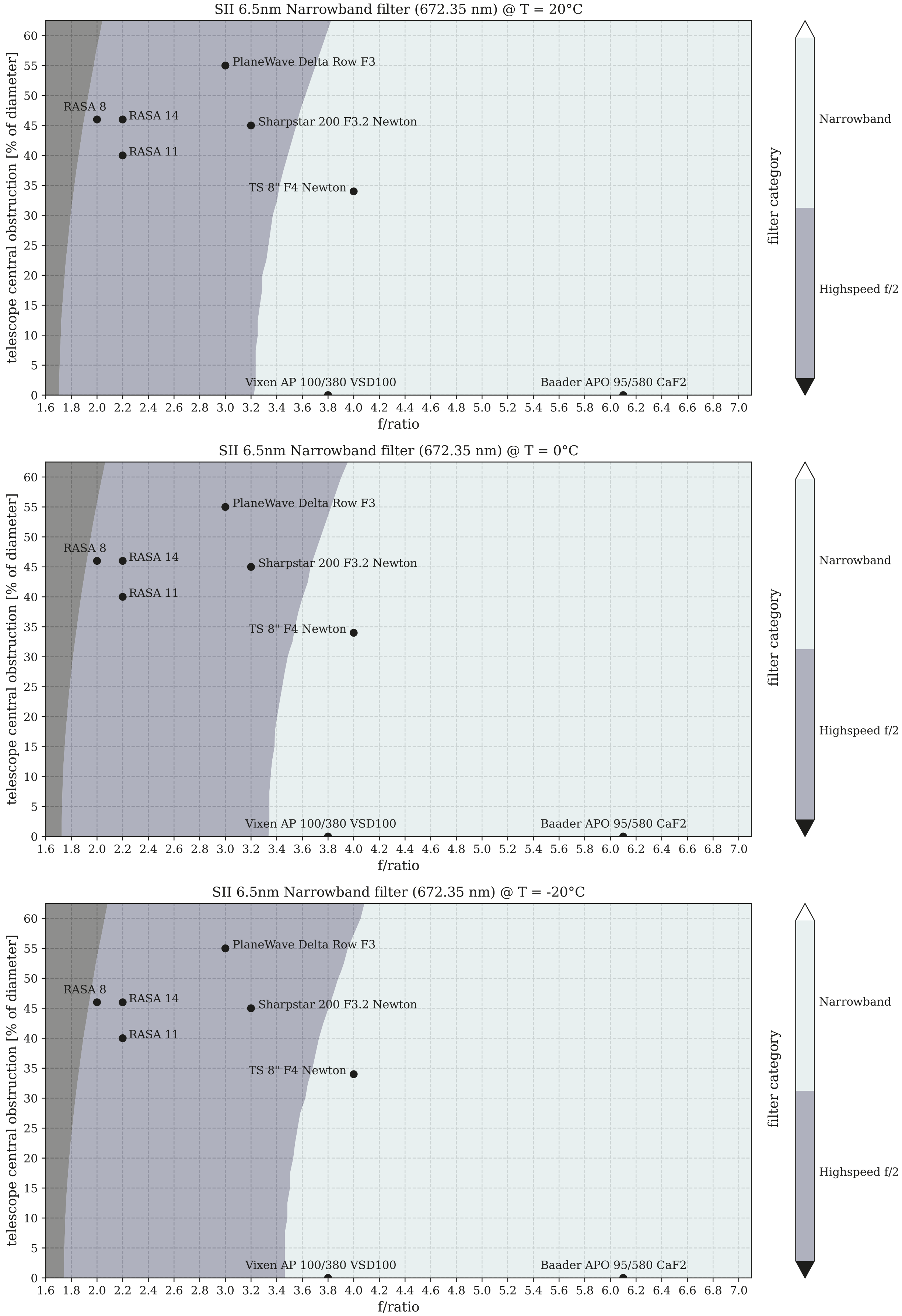


Figure 13: Filter selection guide for **SII 6.5nm Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

5.3 OIII 6.5nm Ultra-Narrowband Filter Selection

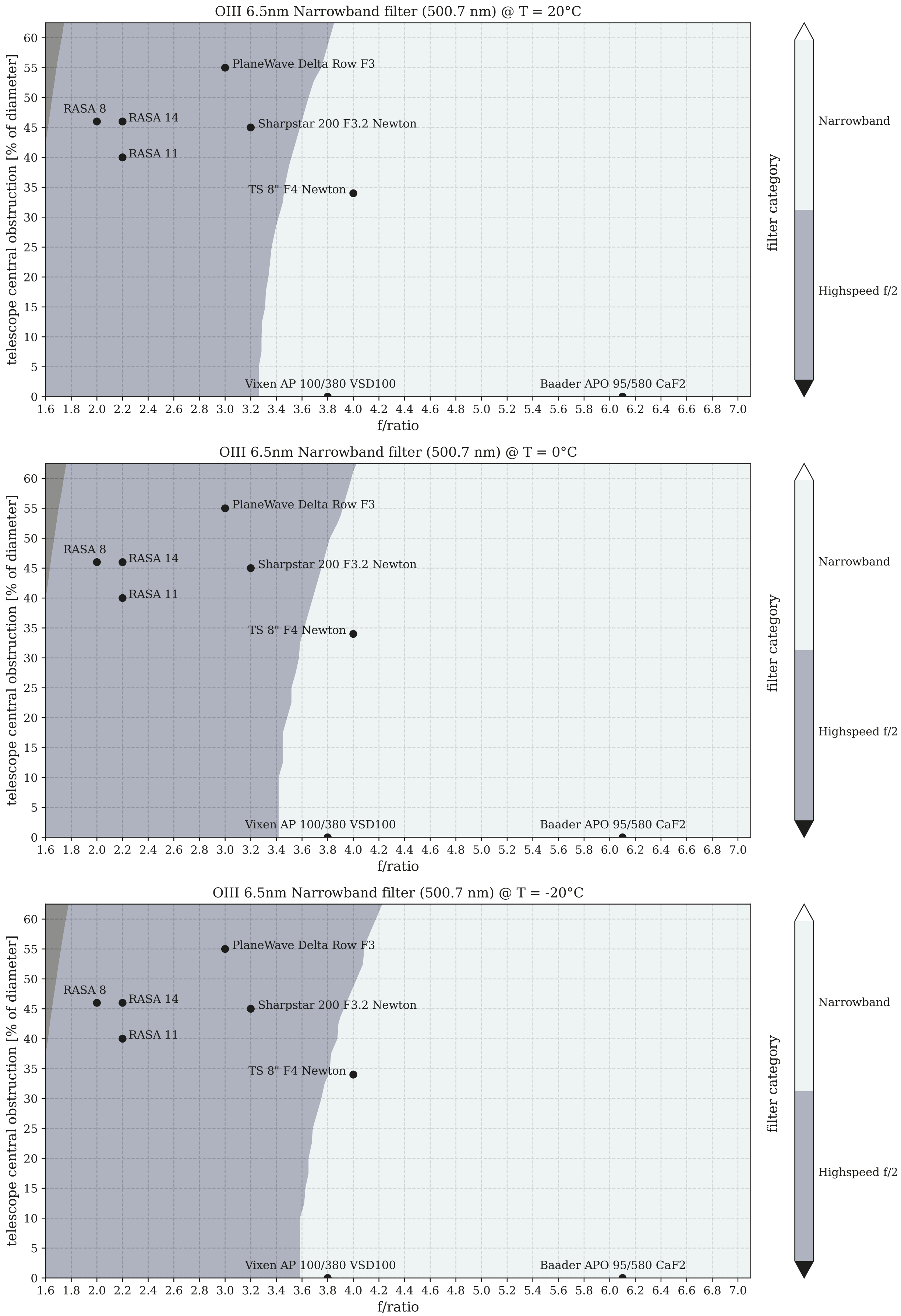


Figure 14: Filter selection guide for **OIII 6.5nm Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

5.4 H-Alpha 3.5nm Ultra-Narrowband Filter Selection

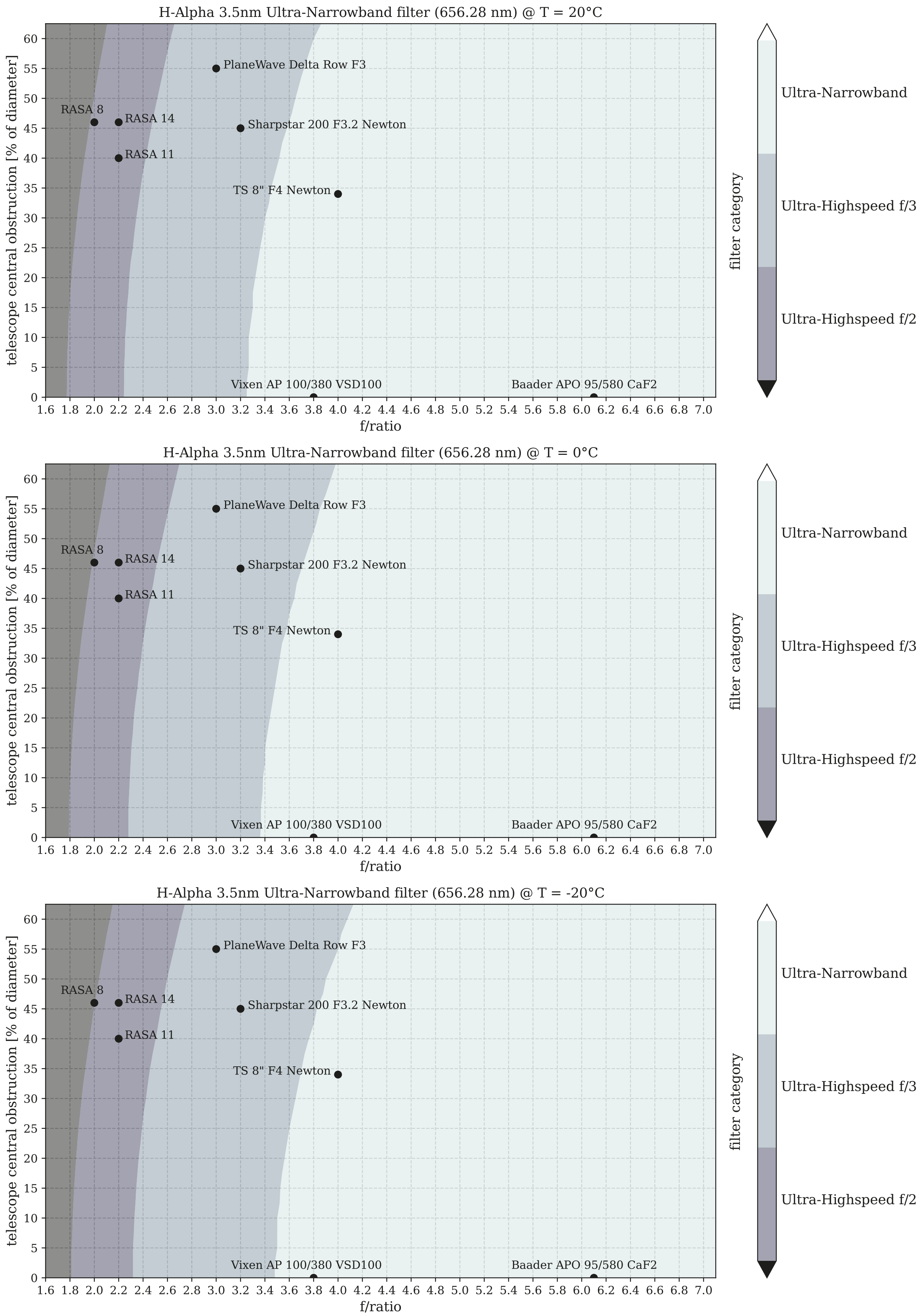


Figure 15: Filter selection guide for **H-Alpha 3.5nm Ultra-Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

5.5 SII 4nm Ultra-Narrowband Filter Selection

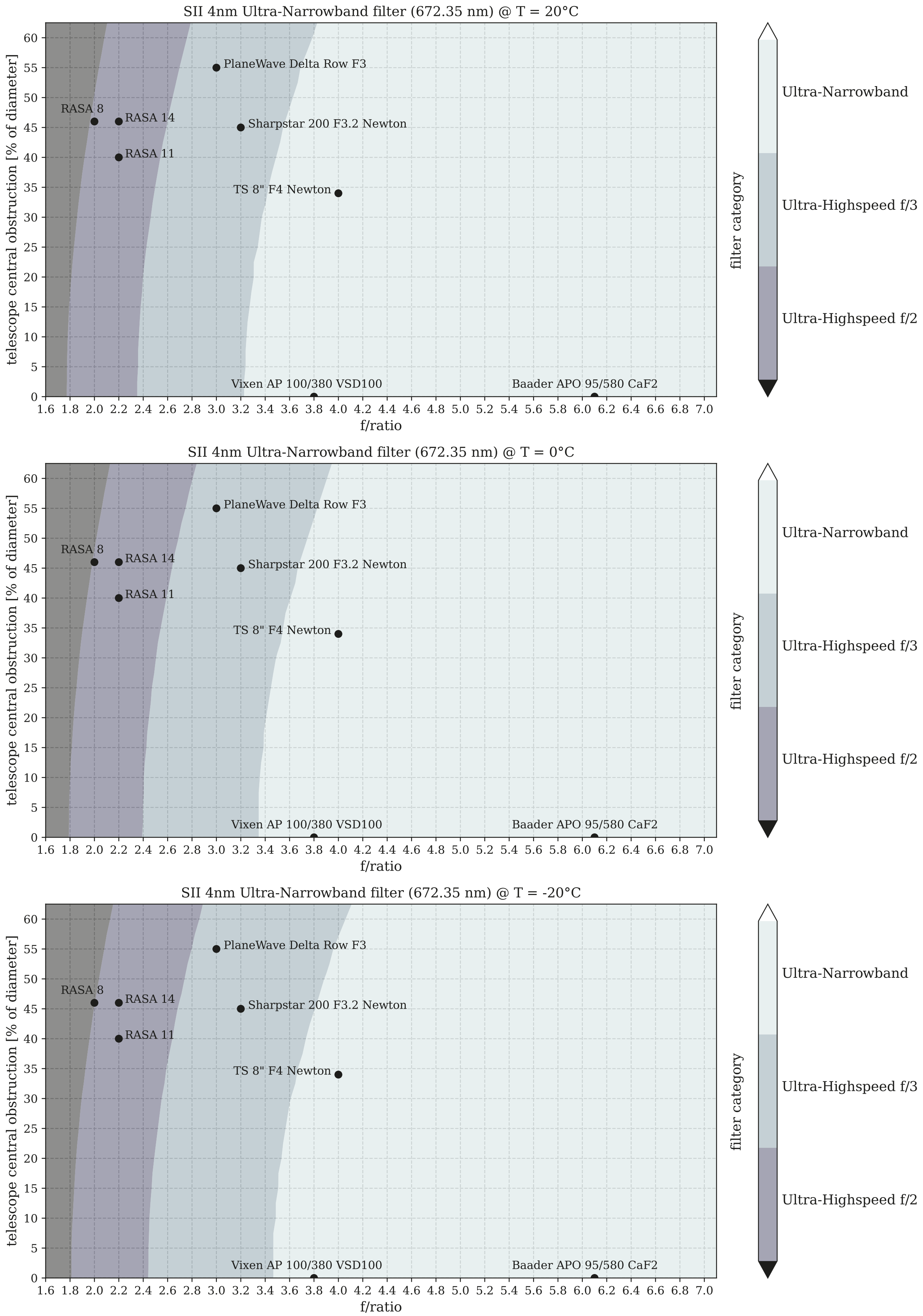


Figure 16: Filter selection guide for **SII 4nm Ultra-Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

5.6 OIII 4nm Ultra-Narrowband Filter Selectionl

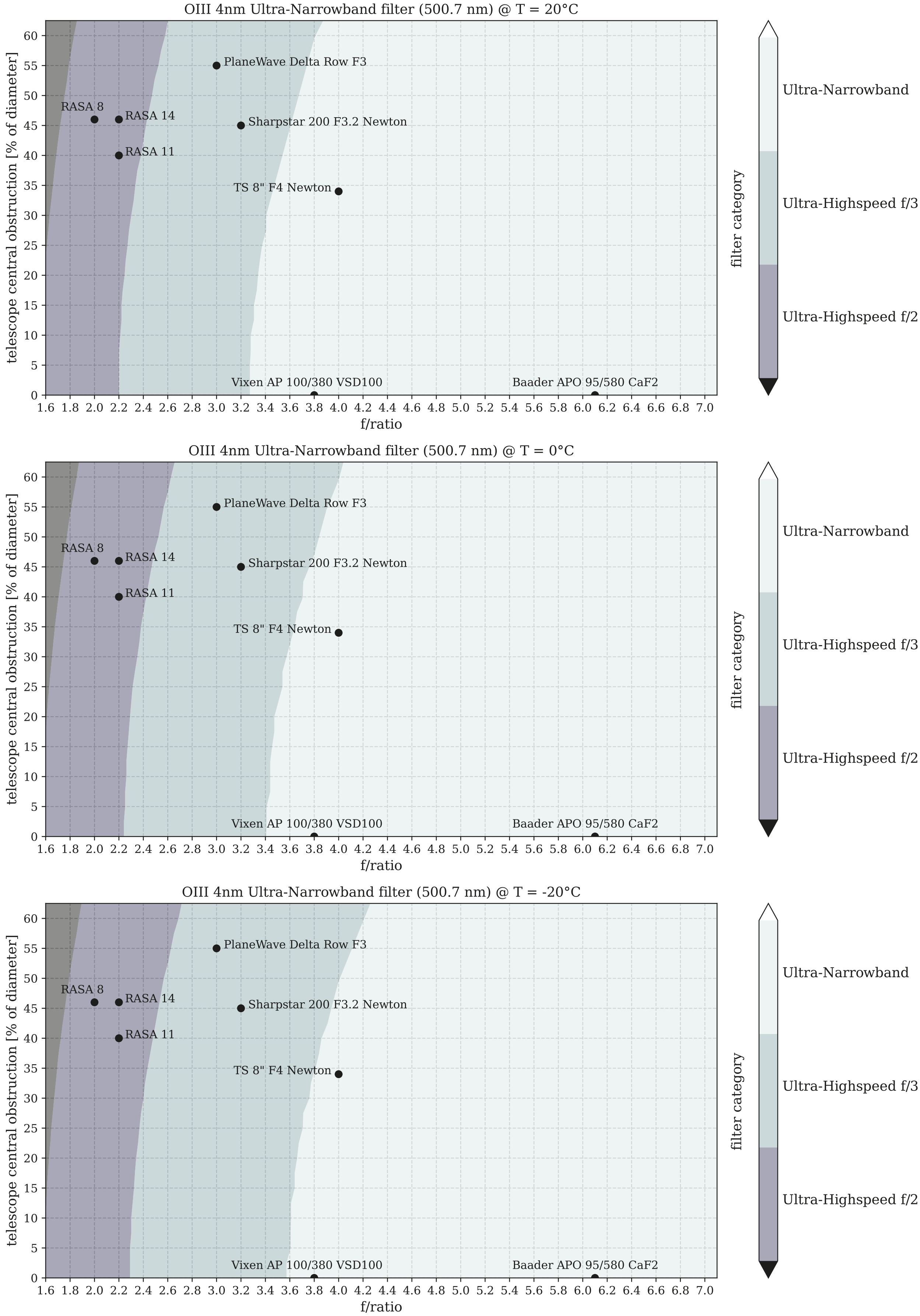


Figure 17: Filter selection guide for **OIII 4nm Ultra-Narrowband** filters at 20°C, 0°C and -20°C ambient temperature. X-axis = focal ratio $f/\#$. Y-axis = central obstruction of the telescope in percent of the diameter, not the area! The coloured areas show the optimal application range of the respective filters (see legend on the right).

A SNR Comparison of Narrowband Filters

Comparison of the signal-to-noise ratio of two narrowband filters under identical observation conditions.

Signal (of the object of interest)	S
Light-pollution signal level (*light-pollution / skyfog*) at wavelength λ	$L(\lambda) = const.$
Half-width filter A bandpass	$FWHM_A$
Half-width filter B bandpass	$FWHM_B$

Assumptions:

- Dark current (DC) and readout noise are neglected as they are identical for both filters.
- The light pollution signal level $L(\lambda)$ is assumed to be constant for all wavelengths in the range of the filter bandpass.
- The object signal is normalised to 1.0.
- The same bandpass shape is assumed for both filters. The light pollution signal can then be assumed to be the signal level multiplied by the half-width: $LS = L$ x $FWHM$.

$$SNR_{FilterA} = \frac{S_{FilterA}}{\sqrt{S_{FilterA} + LS}} = \frac{S_{FilterA}}{\sqrt{S_{FilterA} + L\ FWHM_{FilterA}}} \tag{6}$$

$$SNR_{FilterB} = \frac{S_{FilterB}}{\sqrt{S_{FilterB} + L\ FWHM_{FilterB}}} \tag{7}$$

ratio of the two SNR:

$$\frac{SNR_{FilterA}}{SNR_{FilterB}} = \frac{\frac{S_{FilterA}}{\sqrt{1+L\ FWHM_{FilterA}}}}{\frac{S_{FilterB}}{\sqrt{1+L\ FWHM_{FilterB}}}} = \frac{S_{FilterA}\sqrt{1+L\ FWHM_{FilterB}}}{S_{FilterB}\sqrt{1+L\ FWHM_{FilterA}}} \tag{8}$$

Mit $S = 1$:

$$\frac{SNR_{FilterA}}{SNR_{FilterB}} = \frac{\sqrt{1+L\ FWHM_{FilterB}}}{\sqrt{1+L\ FWHM_{FilterA}}} \tag{9}$$

It can be seen that the SNR ratio depends on the level of light pollution. Figure 18 on the next page shows an example of the SNR comparison of a **3.5nm Ultra-Narrowband** to a **6.5nm Narrowband** filter:

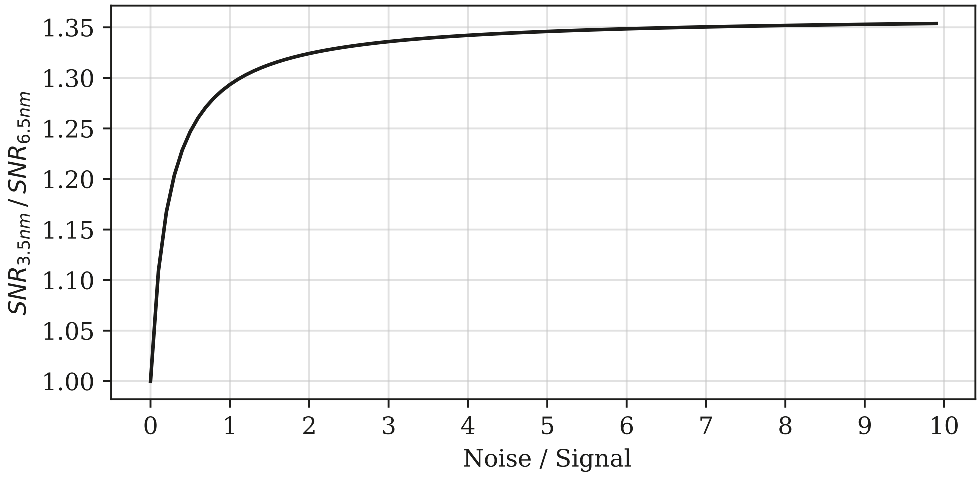


Figure 18: Verhältnis der SNRs

Figure 19: SNR comparison of a 3.5nm Ultra-Narrowband (filter A) to a 6.5nm Narrowband Filter (filter B): $\frac{SNR_{3.5nm}}{SNR_{6.5nm}}$. The ratio converges towards ≈ 1.36 with increasing light pollution level.

The root function converges towards a limit value:

$$\lim_{x\to\infty} f(L) = \lim_{x\to\infty} \sqrt{\frac{1 + L \cdot FWHM_{FilterB}}{1 + L \cdot FWHM_{FilterA}}} = \sqrt{\lim_{x\to\infty} \frac{1 + L \cdot FWHM_{FilterB}}{1 + L \cdot FWHM_{FilterA}}} \tag{10}$$

$$= \sqrt{\lim_{x\to\infty} \frac{\frac{1}{L} + FWHM_{FilterB}}{\frac{1}{L} + FWHM_{FilterA}}} = \sqrt{\frac{FWHM_{FilterB}}{FWHM_{FilterA}}}$$

In the case of the comparison of **3.5nm Ultra-Narrowband** to **6.5nm Narrowband** filters:

$$\frac{SNR_{FWHM=3.5nm}}{SNR_{FWHM=6.5nm}} = \sqrt{\frac{6.5}{3.5}} = 1.363... \tag{11}$$

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